

**Paper Reference 9FM0/3A**  
**Pearson Edexcel**  
**Level 3 GCE**

**Further Mathematics**  
**Advanced**  
**PAPER 3A: Further Pure**  
**Mathematics 1**

**Wednesday 19 June 2024 – Afternoon**

**Time: 1 hour 30 minutes**

**YOU MUST HAVE**  
**Mathematical Formulae and Statistical**  
**Tables (Green), calculator**

**YOU WILL BE GIVEN**  
**Diagram Booklet**  
**Answer Booklet**

**Y75686RA**

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## **INSTRUCTIONS**

**In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.**

**Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.**

**Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.**

**Do NOT write on this Question Paper.**

**You should show sufficient working to make your methods clear.**

**Answers without working may not gain full credit.**

**Inexact answers should be given to three significant figures unless otherwise stated.**

**Turn over**

## **INFORMATION**

**A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.**

**There are 10 questions in this Question Paper.**

**The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.**

**There may be spare copies of some diagrams in case you need them.**

**Turn over**

**ADVICE**

**Read each question carefully before you start to answer it.**

**Try to answer every question.**

**Check your answers if you have time at the end.**

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**Turn over**

1. (a) Refer to the table for Question 1 in the Diagram Booklet.

Given that

$$y = \ln(3 + x^2)$$

complete the table in the Diagram Booklet with the value of  $y$  corresponding to  $x = 3$ , giving your answer to 4 significant figures.

(1 mark)

(continued on the next page)

Turn over

**1. continued.**

**In part (b) you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

**(continued on the next page)**

**Turn over**

**1. continued.**

**(b) Use Simpson's rule with all the values of  $y$  in the completed table to estimate, to 3 significant figures, the value of**

$$\int_2^5 \ln(3 + x^2) dx$$

**(3 marks)**

**(continued on the next page)**

**Turn over**

**1. continued.**

**(c) Using your answer to part (b)  
and making your method clear,  
estimate the value of**

$$\int_2^5 \ln \sqrt{(3 + x^2)} dx$$

**(1 mark)**

**(Total for Question 1 is 5 marks)**

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**Turn over**

10

2. Use algebra to determine the values of  $x$  for which

$$|x^2 - 2x| \leq x$$

(Total for Question 2 is 4 marks)

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Turn over

3. Use L'Hospital's rule to show that

$$\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = 0$$

(Total for Question 3 is 6 marks)

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Turn over

4. Refer to the information for Question 4 in the Diagram Booklet. It shows the Taylor series expansion of  $f(x)$  about  $x = a$

The curve with equation  $y = f(x)$  satisfies the differential equation

$$\cos x \frac{d^2 y}{dx^2} + y^2 \frac{dy}{dx} + \sin x = 0$$

(continued on the next page)

4. continued.

Given that  $\left(\frac{\pi}{4}, 1\right)$  is a stationary point of the curve,

(a) determine the nature of this stationary point, giving a reason for your answer.

(2 marks)

(b) Show that

$\frac{d^3y}{dx^3} = \sqrt{2} - 2$  at this stationary point.

(4 marks)

(continued on the next page)

Turn over

**4. continued.**

**(c) Hence determine a series solution for  $y$ , in ascending powers of**

**$\left(x - \frac{\pi}{4}\right)$  up to and including the term in**

**$\left(x - \frac{\pi}{4}\right)^3$ , giving each**

**coefficient in simplest form.**

**(2 marks)**

**(Total for Question 4 is 8 marks)**

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**Turn over**

5.  $y = e^{3x} \sin x$

(a) Use Leibnitz's theorem to show that

$$\frac{d^4 y}{dx^4} = 28e^{3x} \sin x + 96e^{3x} \cos x$$

(6 marks)

(continued on the next page)

Turn over

5. continued.

(b) Hence express  $\frac{d^4 y}{dx^4}$  in the form

$$Re^{3x} \sin(x + \alpha)$$

where  $R$  and  $\alpha$  are constants to be determined,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

(3 marks)

**(Total for Question 5 is 9 marks)**

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Turn over

6. The ellipse **E** has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The hyperbola **H** has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where **a** and **b** are positive constants.

(continued on the next page)

Turn over

**6. continued.**

**Given that**

- **the eccentricity of  $H$  is the reciprocal of the eccentricity of  $E$**
- **the coordinates of the foci of  $H$  are the same as the coordinates of the foci of  $E$**

**determine**

**(i) the value of  $a$**

**(ii) the value of  $b$**

**(Total for Question 6 is 6 marks)**

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**Turn over**

- 7. In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

**(continued on the next page)**

7. continued.

(a) Use the substitution

$$t = \tan\left(\frac{\theta}{2}\right) \text{ to show that}$$

$$\int \frac{1}{2\sin\theta + \cos\theta + 2} d\theta$$

$$= \int \frac{a}{(t+b)^2 + c} dt$$

where **a**, **b** and **c** are constants  
to be determined.

(3 marks)

(continued on the next page)

Turn over

7. continued.

(b) Hence show that

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{1}{2\sin\theta + \cos\theta + 2} d\theta = \ln\left(\frac{2\sqrt{3}}{3}\right)$$

(4 marks)

(Total for Question 7 is 7 marks)

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Turn over

8. The parabola **P** has equation  $y^2 = 4ax$ , where **a** is a positive constant.

The point

**A**( $at^2, 2at$ ), where  $t \neq 0$ , lies on **P**

- (a) Use calculus to show that an equation of the tangent to **P** at **A** is

$$yt = x + at^2$$

(3 marks)

(continued on the next page)

Turn over

**8. continued.**

**The point**

**$B(2k^2, 4k)$  and the point**

**$C(2k^2, -4k)$ , where  $k$  is a constant,  
lie on  $P$**

**The tangent to  $P$  at  $B$  and the tangent  
to  $P$  at  $C$  intersect at the point  $D$**

**Given that the area of the triangle  
 $BCD$  is 432**

**(b) determine the coordinates of  $B$   
and the coordinates of  $C$   
(5 marks)**

**(Total for Question 8 is 8 marks)**

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**Turn over**

9. (i) The line  $L_1$  has equation

$$\underline{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$$

The line  $L_2$  has equation

$$\underline{r} = \begin{pmatrix} 13 \\ 5 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are scalar parameters.

(continued on the next page)

Turn over

9. (i) continued.

The lines  $L_1$  and  $L_2$  intersect at the point  $P$

(a) Determine the coordinates of  $P$   
(2 marks)

Given that the plane  $\Pi$  contains both  $L_1$  and  $L_2$

(b) determine a Cartesian equation for  $\Pi$   
(4 marks)

(continued on the next page)

Turn over

**9. continued.**

**(ii) Determine a Cartesian equation for each of the two lines that**

- pass through  $(0, 0, 0)$**
- make an angle of  $60^\circ$  with the  $x$ -axis**
- make an angle of  $45^\circ$  with the  $y$ -axis**

**(4 marks)**

**(Total for Question 9 is 10 marks)**

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**Turn over**

10. The motion of a particle **P** along the **x**-axis is modelled by the differential equation

$$t^2 \frac{d^2 x}{dt^2} - 2t(t + 1) \frac{dx}{dt} + 2(t + 1)x = 8t^3 e^t \quad (\text{I})$$

where **P** has displacement **x** metres from the origin **O** at time **t** minutes,  $t > 0$

(continued on the next page)

10. continued.

- (a) Show that the transformation  $x = tu$  transforms the differential equation (I) into the differential equation

$$\frac{d^2u}{dt^2} - 2 \frac{du}{dt} = 8e^t$$

(4 marks)

(continued on the next page)

Turn over

**10. continued.**

**Given that  $P$  is at  $O$  when**

**$t = \ln 3$  and when**

**$t = \ln 5$**

**(b) determine the particular solution  
of the differential equation (I)  
(8 marks)**

**(Total for Question 10 is 12 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**

**END OF PAPER**

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